NOTIZEN

On a Consequence of Bogoliubov's Functional Ansatz

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The Landau-spectrum in the theory of normal Fermi liquid has been derived for a classical system under two assumptions: Bogoliubov's functional Ansatz and functional expansion Ansatz. A correlationless form of Boltzmann-Landau equation for such a system has also been given.

In recent years [1] much work has been devoted to the problem of deriving an adequate kinetic equation for a dense gas, which describes the irreversible nature of the system and its approach to equilibrium. Some approaches to the problem [2, 3] seem to indicate that the kinetic equation proposed by Landau [4] in connection with the theory of a normal Fermi liquid can be successfully applied to derive the transport coefficients of a moderately dense quantum gas at a temperature higher than the critical temperature of degeneracy. A direct derivation of Landau's equation for such a quantum system has been recently presented by Baerwinkel and Grossman [5] where they assume the long wave length limit and lowest order in the scattering amplitude. However, the Prigogine school [6] has extended Landau's quasi-particle (QP) concept to classical systems. This method is based on perturbation theory and the asymptotic limit approach to equilibrium. We shall show in this paper how the collisionless form of the Landau equation may be derived in a formal manner from Bogoliubov's Ansatz for a classical system of N particles enclosed in a volume V interacting with a two-body potential Φ. However the problem of approach to equilibrium and existence of QP's has not been discussed in this note. Rather, these results have been interpreted in such a way as to agree with the definition of QP's. (See e. g., Ref. [7].) The problem of approach wil be dealt with in a separate publication.

Our method is based on two assumption (for notation and details see Ref. [1]): (i) Bogoliubov's functional Ansatz, which states that after a time which is longer than the duration of a single collision but short compared to the time between successive collisions, the time dependence of the s-particle distribution F_s is functionally dependent on the one-particle distribution F_1. Then F_s can be written as

$$F_s(X_1,\dots,X_s\,|\,F_1)=\prod_{i=1}^{s}(F_1(X_i))P_s(X_1,\dots,X_s\,|\,F_1). \tag{1}$$

where $X_i=(\boldsymbol{r}_i,p_i)$ describe the particle positions in phase space. (ii) functional expansion ansatz, that a functional of the form $K(X_1X_2\,|\,F_1)$ can be expanded, in the thermodynamic limit, as

$$K(X_1X_2\,|\,F_1)=\sum\frac{n^\nu}{\nu!}\cdot\int K_\nu(X_1X_2\,|\,Y_1\dots Y_\nu)\prod_{i=1}^{\nu}(F_1(Y_i)\,\mathrm{d}Y_i). \tag{2}$$

This corresponds to the assumption of a virial expansion. We shall hereafter use the normalized distribution F_N. Then by definition [1]

$$F_s(X_1,\dots,X_s\,|\,F_1)=V^s\int F_N(X_1\dots X_N)\,\mathrm{d}X_{s+1}\dots\mathrm{d}X_N$$
$$\cdot\int P_s(X_1,\dots,X_s\,|\,F_1)\prod_{i=1}^{s}(F_1(X_i)\,\mathrm{d}X_i)=V^s. \tag{3}$$

Then the total energy E (defined as the average of the Hamiltonian

$$H=\sum\frac{p^2}{2m}+\sum_{i<k}\Phi_{ik}$$

over the distribution F_N) can be written, in the thermodynamic limit, as

$$E=n\int\frac{p^2}{2m}F_1(X,t)\,\mathrm{d}X+\frac{n^2}{2}\cdot\int\Phi(|\boldsymbol{r}_1-\boldsymbol{r}_2|)\,F_2(X_1,X_2,t)\,\mathrm{d}X_1\,\mathrm{d}X_2 \tag{4}$$

$(n=\lim N/V,\ V\to\infty,\ N\to\infty)$.

Though Eq. (4) is exact, nothing is practically achieved without special assumptions. In the present case we shall use the assumptions (i) and (ii). Assumption (i) will make E a functional of F_1 only, whereas assumption (ii) enables us to develop $E(F_1)$ in an explicit functional series in F_1. The pair correlation $P_2(X_1X_2\,|\,F_1)$ in the expression of F_2 in Eq. (1) takes also the multibody processes into account through its functional dependence on F_1. If we define

$$K=\tfrac{1}{2}\,\Phi(|r_1-r_2|)\,P(X_1,X_2\,|\,F_1)$$

in (2) then $E(F_1)$ can be developed in a functional series with kernels K_ν and thus all K_ν are symmetric

[1] See e. g. Ta-You Wu, Kinetic Equations of Gases and Plasmas, Addison-Wesley Publishing Company, 1966.
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in X_1 and X_2. One could also assume that K_ν be symmetric under the interchange of any two Y's in its argument. But $K_\nu(X_1 X_2 | Y_1 \dots Y_\nu)$ may not in general be symmetric under the interchange of any one of X_1 or X_2 with one of $Y_1, \dots, Y_\nu$. However, we shall be interested in $E(F_1)$, in which the integrations have to be over all the variables $(X_1 X_2 Y_1 \dots Y_\nu)$ in K_ν. Then we may use, instead of K_ν, a symmetrized kernel $H_\nu(X_1 X_2 Y_1 \dots Y_\nu)$ as well without affecting $E(F_1)$ in Eq. (4) at all. We shall take,

$$H_s(X_1 \dots X_s) = \frac{1}{s!} \sum_{\text{all permutations}} P\, K_s(X_1 X_2 X_3 \dots X_s) \tag{5}$$

We further note from Eq. (1) that $\int F_1(X)\, dX = V$, so that $\int f(X)\, dX = N$ where $f = n F_1$. Thus, in terms of f, $E(f)$ takes the functional form,

$$E(f) = \sum_s \frac{1}{s!} \int H_s(X_1, X_2, \dots, X_s) \prod_{i=1}^{s} f(X_i)\, dX_i, \tag{6}$$

where $H_1(X_i) = \dfrac{p_i^2}{2m}$.

$E(f)$ contains all interactions through H_s. The result of Eq. (6) can be interpreted as follows: A system of N-particles interacting with a two-body potential is equivalent in the thermodynamic limit to a system with an equal number of particles interacting via multibody forces H_s and whose one-particle distribution is the same as that for the original system, the s-particle distributions being uncorrelated. This equivalent system is the classical analog of LANDAU's QP ensemble. The QP's interact with multibody forces which are momentum dependent, the Hamiltonian of this system being, in accordance with Eq. (6),

$$H = \sum_i H_i(i) + \sum_{i<\nu} H_2(i\,\nu) + \sum_{i<\nu<k} H_3(i\,\nu\,k) + \dots \tag{7}$$

Thus, there is a one-to-one correspondence between the N QP's described by correlationless distributions with the original system of N particles. The quasiparticle energy $\varepsilon(X)$ is defined by [4]

$$\delta E = \int \varepsilon(X_1)\, \delta f(X_1)\, dX_1$$

where from Eq. (6) we have

$$\varepsilon(X) = \frac{p^2}{2m} + \sum_s \frac{1}{s(s-1)!} H_s(X_1 \dots X_s) \prod_{i=2}^{s} (f(X_i)\, dX_i). \tag{8}$$

This result is also similar to that obtained by MORITA et al. for a quantum system [8]. Before deriving a kinetic equation for $f(X)$, we would like to point out how the various quantities $\varepsilon(X)$ and H_s can be determined phenomenologically. It can be shown [2] that the interactions H_s are related to the virial coefficient in equilibrium theory [2]. For example, in the binary collision approximation, $\varepsilon(X)$ can be written as

$$\varepsilon(X) = \frac{p^2}{2m} + \int H_2(X, X_1)\, f(X_1)\, dX_1$$

where H_2 can be determined from the second virial coefficient [2]

$$B(T) = \frac{1}{2kT} \int H_2(X, X_1)\, f_0(X)\, f_0(X_1)\, dX\, dX_1$$

where $f_0(X)$ is the normalized equilibrium distribution. Similarly the higher virial coefficients will be related (in addition to the binary collision term) to higher order collision terms. The correlationless form of Landau's kinetic equation for $f(X)$ can now be easily deduced from the Liouville's equation for the quasi-particle system interacting with multibody explicitly momentum dependent forces. Replacing the s-particle distribution by the product of one-particle distributions, one readily obtains the kinetic equation

$$\frac{\partial f}{\partial t} + \frac{\partial \varepsilon}{\partial \boldsymbol{p}} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - \frac{\partial \varepsilon}{\partial \boldsymbol{r}} \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0. \tag{9}$$

Such an equation has also been derived by KADANOFF and BAYM for a quantum system using Green's function technique [9]. On the other hand, based on the Wigner distribution and von Neumann's equation, BAERWINKEL and GROSSMAN have derived the Landau theory under certain assumptions which are contained explicitly in their work [5]. They obtained the general solution for the quantum two-particle distribution F_2, and then using the functional Ansatz derived the Boltzmann-Landau equation. (For details and further references see Ref. [5].)

The natural extension of (9) to include the collision integral for a classical system will be the subject of further investigation. The results of such investigations will be presented at a later date.

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